Physical Optics AFTA image formation

The role of vector waves in AFTA PSF

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- Dark energy data set requirements
  - System PSF EE50% radius, 325 mas at 2 microns
- WL data set requirements
  - PSF second moment $(I_{xx} + I_{yy})$ known to a relative error $\leq 9.3\text{x}10^{-4}$ rms (shape/color filters only)
  - PSF ellipticity $(I_{xx} - I_{yy}, 2 \cdot I_{xy}) / \left(I_{xx} + I_{yy}\right)$ known to a relative error of $\leq 4.7x10^{-4}$ rms (shape/color filters only)
  - System PSF EE50% radius

$$\leq 166 \text{ (J band)}, 185 \text{ (H) or } 214 \text{ (K) mas}$$

Vector wave optics determines the shape of the PSF

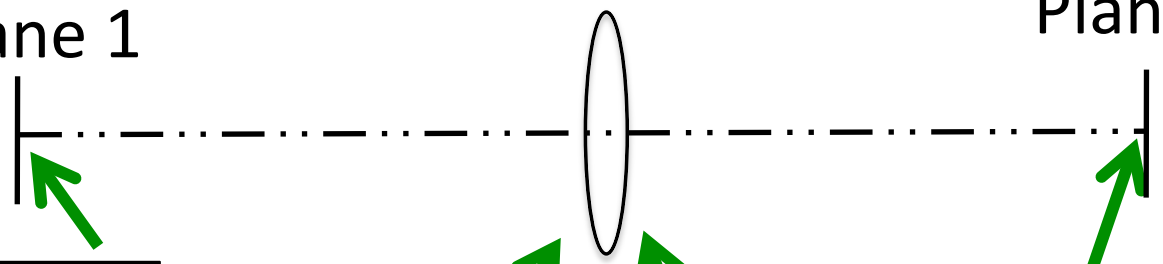
Outline

- Physics of image formation
  - Complements ray-tracing
- Partial coherence, vector waves & polarization
- Secondary structure shadows the pupil
- Sources of amplitude errors
- Sources of phase errors
- References

Physics of image formation

- An optical system operates on both the incoming phase & amplitude to form an image at a focal plane
- In general each optical element introduces its own amplitude and phase errors
- The image is the modulus squared of the sum of the amplitudes and phases at the image plane

E-M Field Propagation



The field
at plane 1

$$U_1(x_1,y_1) = a_1(x_1,y_1) + ib_1(x_1,y_1)$$

The field just
to the LEFT of
plane 2

$$U_2^-(\xi_2,\eta_2) = a_2(\xi_2,\eta_2) + ib_2(\xi_2,\eta_2)$$

The field just
to the RIGHT of
plane 2 EXIT pupil

$$U_2^+(\xi_2,\eta_2) = U_2^-(\xi_2,\eta_2)\boxed{\left[c_2(\xi_2,\eta_2) + id_2(\xi_2,\eta_2)\right]_T}$$

More

The field at the
image plane 3

$$U_3(x_3,y_3) = a_3(x_3,y_3) + ib_3(x_3,y_3)$$

The focal plane records INTENSITY

$$I_3(x_3,y_3) = \left|U_3(x_3,y_3)\right|^2$$

Object, Pupil and Image Planes

J. Goodman and M. Born & E. Wolf use the Huygens-Fresnel principal and the Fresnel and Fraunhofer approximations to show that

$$I_3(x_3,y_3) = A^2 \left| \int^{\infty}\int^{\infty} U_2^+(\xi_2,\eta_2) \underbrace{\exp\left[-i\frac{2\pi}{\bar{\lambda} f}(x_3\xi_2 + y_3\eta_2)\right]}_{\text{Powered optical element}} \right|^2$$

f = focal length

$\bar{\lambda}$ = quasi-monochromatic wavelength

A is a scaling factor

$U_2^+(\xi_2,\eta_2)$ is complex transmittance of the pupil.

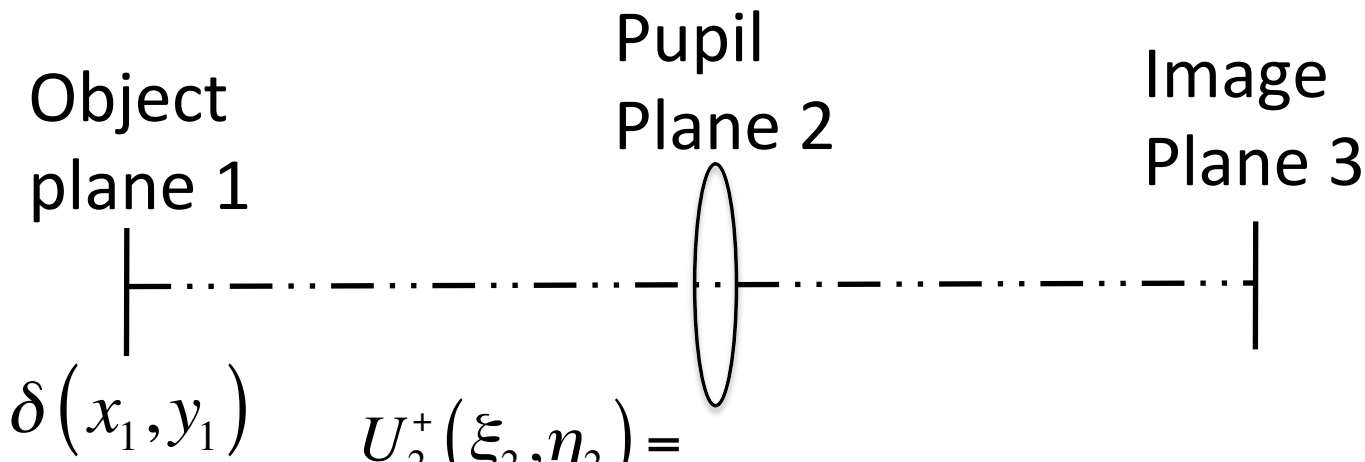
$$U^+(\xi_2,\eta_2) = \begin{cases} 1 \text{ inside the aperture} \\ 0 \text{ otherwise} \end{cases}$$

<u>Ideal telescope</u>

$$U^+(\xi_2,\eta_2) \neq \begin{cases} 1 \text{ inside the aperture} \\ 0 \text{ otherwise} \end{cases}$$

<u>Real-world telescope</u>

The PSF



$$U_2^+\left(\xi_2, \eta_2\right) =$$

$$U_2^-\left(\xi_2, \eta_2\right)\left[c_2\left(\xi_2, \eta_2\right) + id_2\left(\xi_2, \eta_2\right)\right]_T$$

$$I\left(x_3, y_3\right) =$$

$$\left|\delta(x_3, y_3) \otimes \mathbf{F}\ \left\{\left[c_2\left(\xi_2, \eta_2\right) + id_2\left(\xi_2, \eta_2\right)\right]_T\right\}\right|^2$$

Therefore the function: $\left[c_2\left(\xi_2, \eta_2\right) + id_2\left(\xi_2, \eta_2\right)\right]_T$ determines the shape of the PSF

Exit pupil transmittance $\left[c_2\left(\xi_2,\eta_2\right)+id_2\left(\xi_2,\eta_2;\lambda\right)\right]_T$

Sources of wavefront distortions

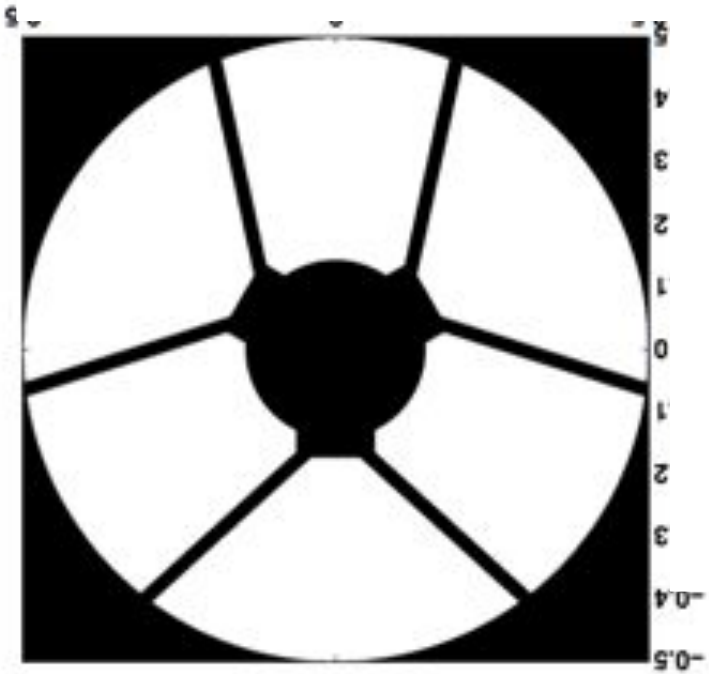
$c_2\left(\xi_2,\eta_2\right)$ Amplitude

- **Secondary and support structure masks (vignettes) portions of the exit pupil as a function of FOV.**
- Cosine θ reflection drop off as a function of pupil radius because of the convex primary mirror

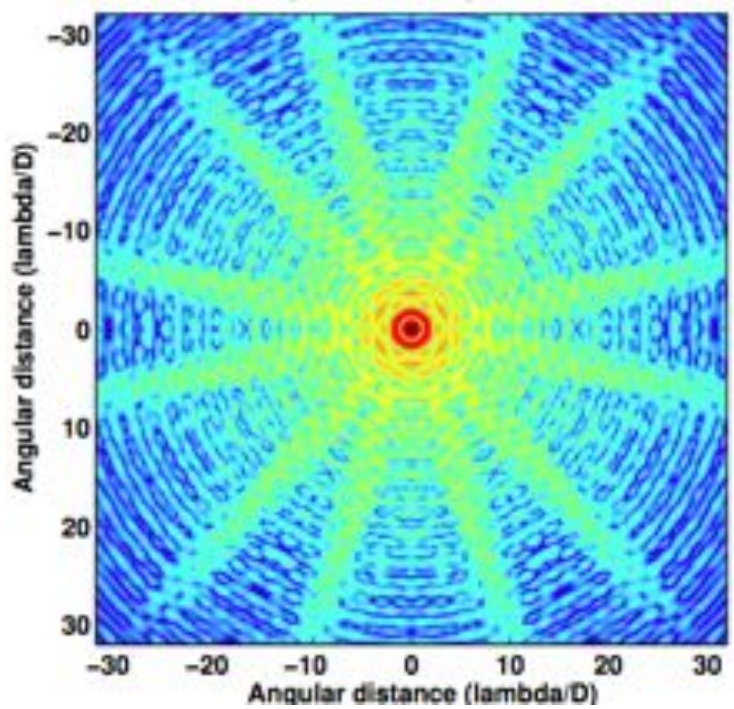
$d_2\left(\xi_2,\eta_2\right)$ Phase

- **Mirror figure errors change with field position**
- Polarization apodization
- Optical thin film inhomogeneities

Exit Pupil Amplitude mask



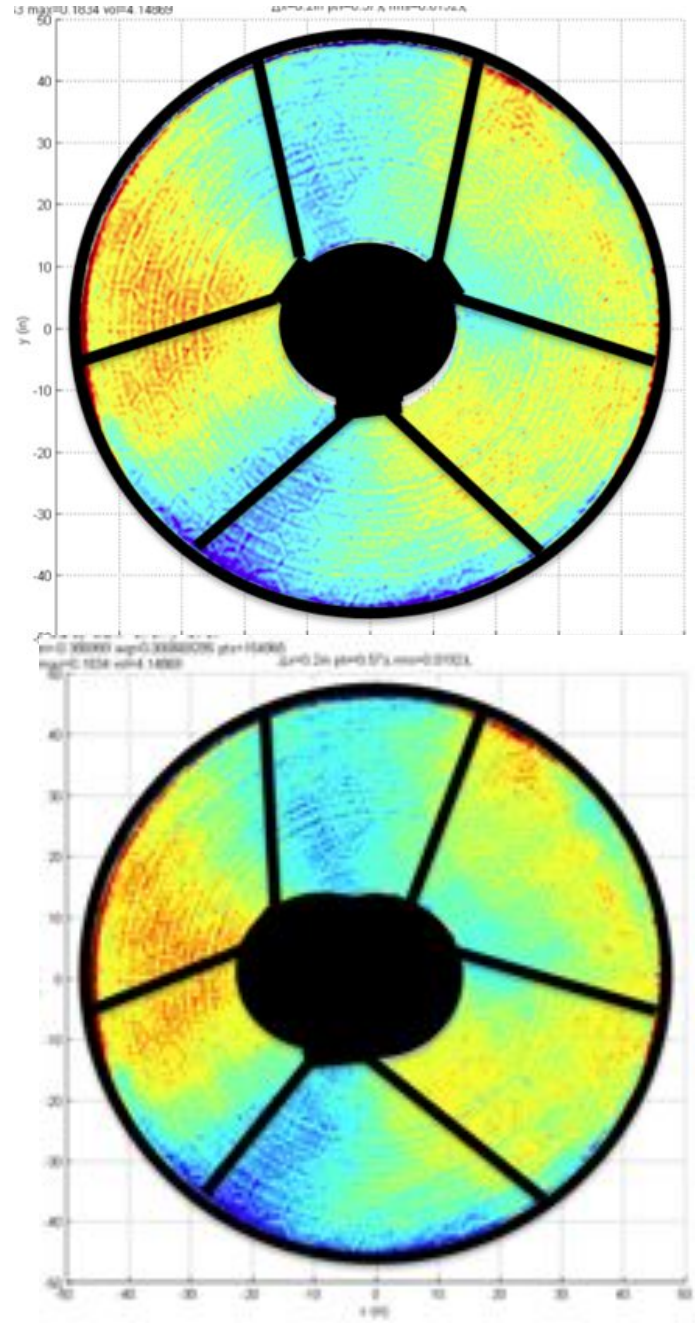
PSF (from J. Kasdin)



- Left: exit pupil amplitude mask
- Right: Point spread function (J. Kasdin)

Exit pupil map

- The exit pupil of the telescope – mirror is masked by secondary support system
- Top: on axis
- Bottom: off axis at a field point
- Amplitude and phase at image plane are different
- => PSF is field dependent
- Isoplanatic patch = sub-area of the image plane over which the image formation is a “linear” process



Exit pupil transmittance $\left[c_2\left(\xi_2,\eta_2\right)+id_2\left(\xi_2,\eta_2;\lambda\right)\right]_T$

Sources of wavefront distortions

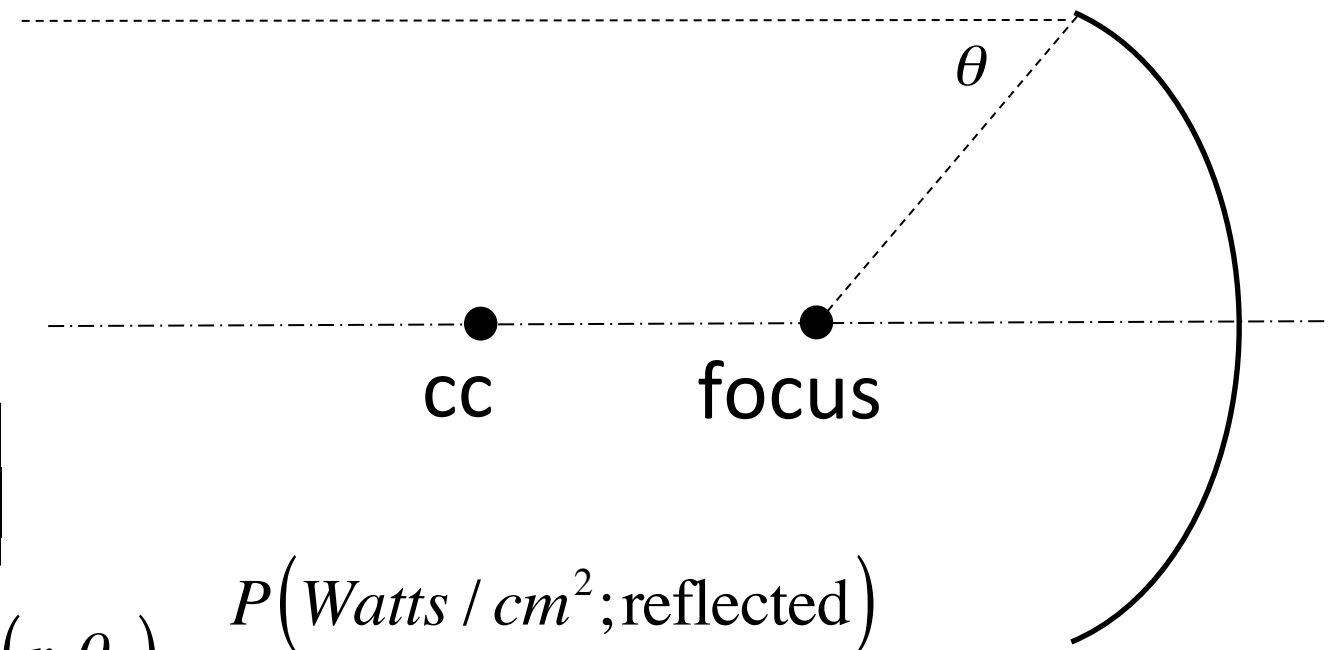
$c_2\left(\xi_2,\eta_2\right)$ Amplitude

- Secondary and support structure masks (vignettes) portions of the exit pupil as a function of FOV.
- **Cosine θ reflection drop off as a function of pupil radius because of the convex primary mirror**

$d_2\left(\xi_2,\eta_2\right)$ Phase

- Mirror figure errors change with field position
- Polarization apodization
- Optical thin film inhomogeneities

Amplitude reflectivity of AFTA



$$\theta = \arctan\left[\frac{0.5}{F^{\#}}\right]$$

$$R(r,\theta) = \frac{P\left(Watts/cm^2;\text{reflected}\right)}{P\left(Watts/cm^2;\text{incident}\right)}$$

$$R(r,\theta) = R_N \cos\theta = 0.92 R_N$$

Necessary condition for image formation: coherence

High quality images require that EM radiation coming from **all** points on the pupil interfere constructively or destructively at all points across the image plane.

An example of incoherent light is if two beams are orthogonally polarized – no interference & scattered light is increased

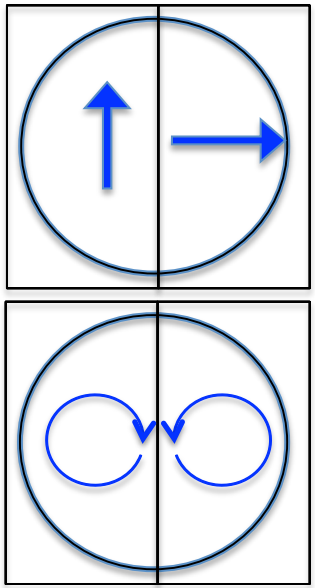
Two beams are partially coherent if portions of their beams are orthogonally polarized

Polarization and PSF: Gedanken experiment

Place two sheets of polarizers, each orthogonal to the other, over the pupil of a telescope.

The point spread function is no longer the Bessel function

$$PSF = I\left(r_3, \theta_3\right) \neq \left[\frac{2J_1(r_3)}{r_3}\right]^2$$



It is the linear superposition of the PSFs for two D shaped pupils placed back to back.

ANY surface in the entire optical path that introduces partial polarization will distort the $PSF(x_3, y_3)$ and increase scattered light.

Exit pupil transmittance $\left[c_2\left(\xi_2,\eta_2\right)+id_2\left(\xi_2,\eta_2;\lambda\right)\right]_T$

Sources of wavefront distortions

$c_2\left(\xi_2,\eta_2\right)$ Amplitude

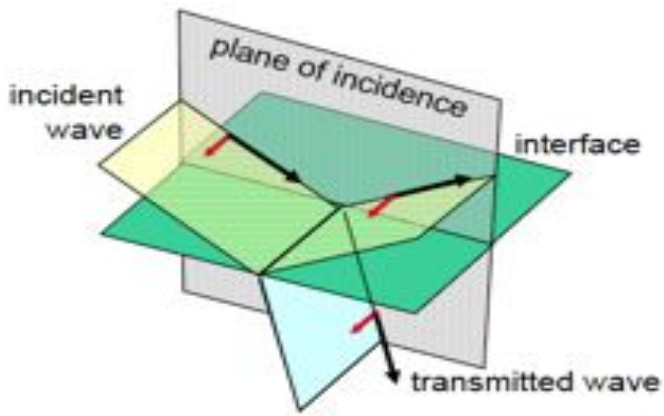
- Secondary and support structure masks (vignettes) portions of the exit pupil as a function of FOV.
- Cosine θ reflection drop off as a function of pupil radius because of the convex primary mirror

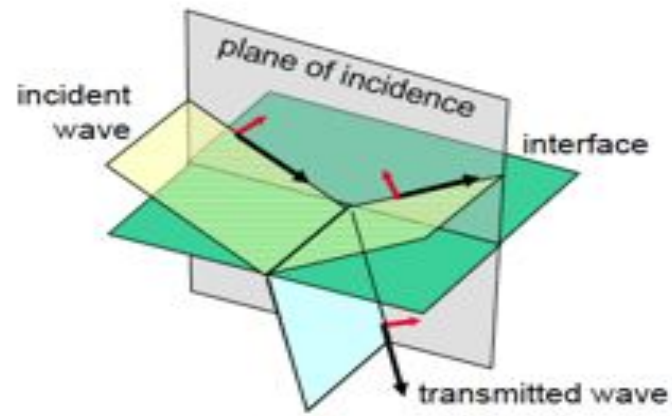
$d_2\left(\xi_2,\eta_2;\lambda\right)$ Phase

- Mirror figure errors and core structure print through change with field position
- **<u>Polarization apodization</u>**
- **<u>Optical thin film inhomogeneities</u>**

For dielectric substrates

Reminder: the Fresnel equations





s-polarized light:

$$r_\perp = \frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

$$t_\perp = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

p-polarized light:

$$r_\parallel = \frac{n_i \cos(\theta_t) - n_t \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

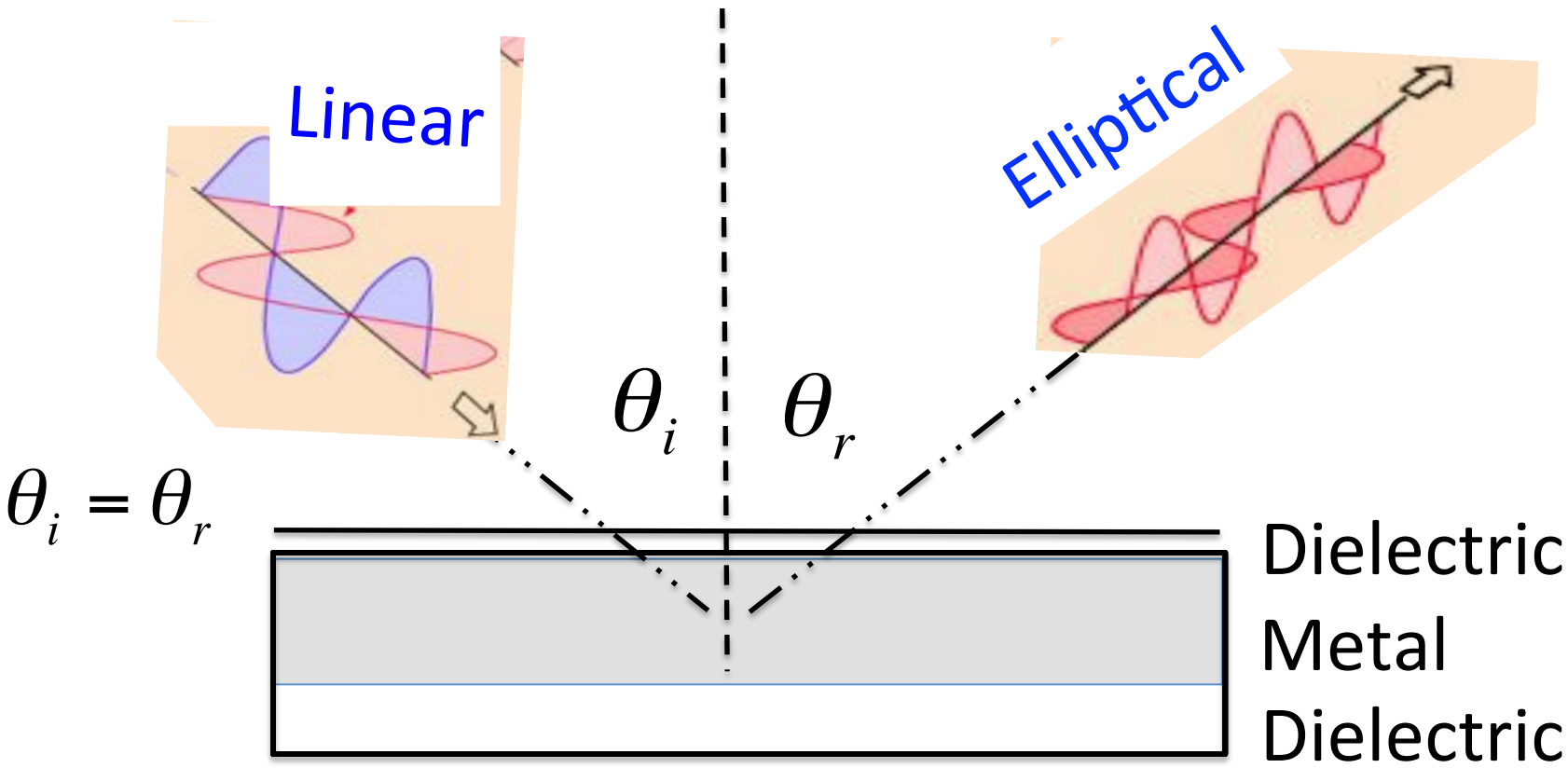
$$t_\parallel = \frac{2 n_i \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}$$

And, for <u>both</u> polarizations: $n_i \sin(\theta_i) = n_t \sin(\theta_t)$

For metal substrates it is more complicated

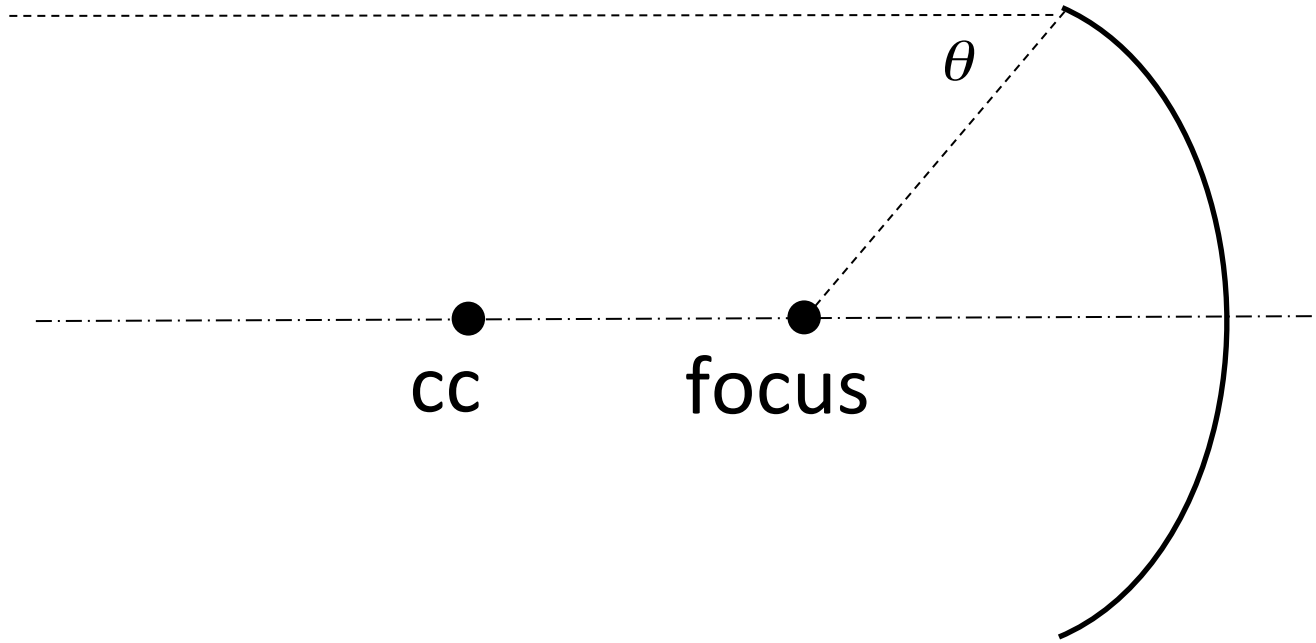
$$\hat{n}(\lambda)_t \sin\theta_t = \hat{n}(\lambda)_i \sin\theta_i \quad \text{where } \hat{n}(\lambda) = n(\lambda)\left[1 - ik(\lambda)\right]$$

Reflection at an angle from a highly reflecting metal mirror changes the phase of the complex wave. –The wave penetrates the metal



Amplitude and phase of the elliptically polarized light depends on θ_i and $\hat{n}$

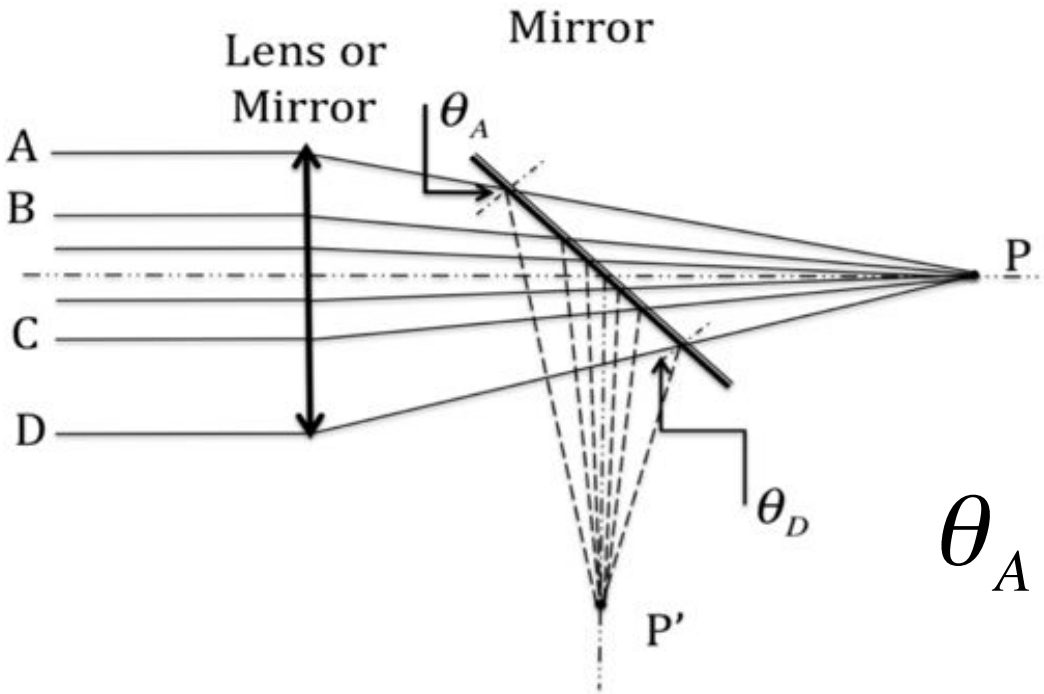
Sources of polarization in F# 1.2 AFTA



Reflection from a tilted metal thin film.
Marginal ray is deviated by angle θ .

$$\theta = 22.62 \text{ degrees}$$

Sources of polarization in AFTA



Flat mirror in a converging beam

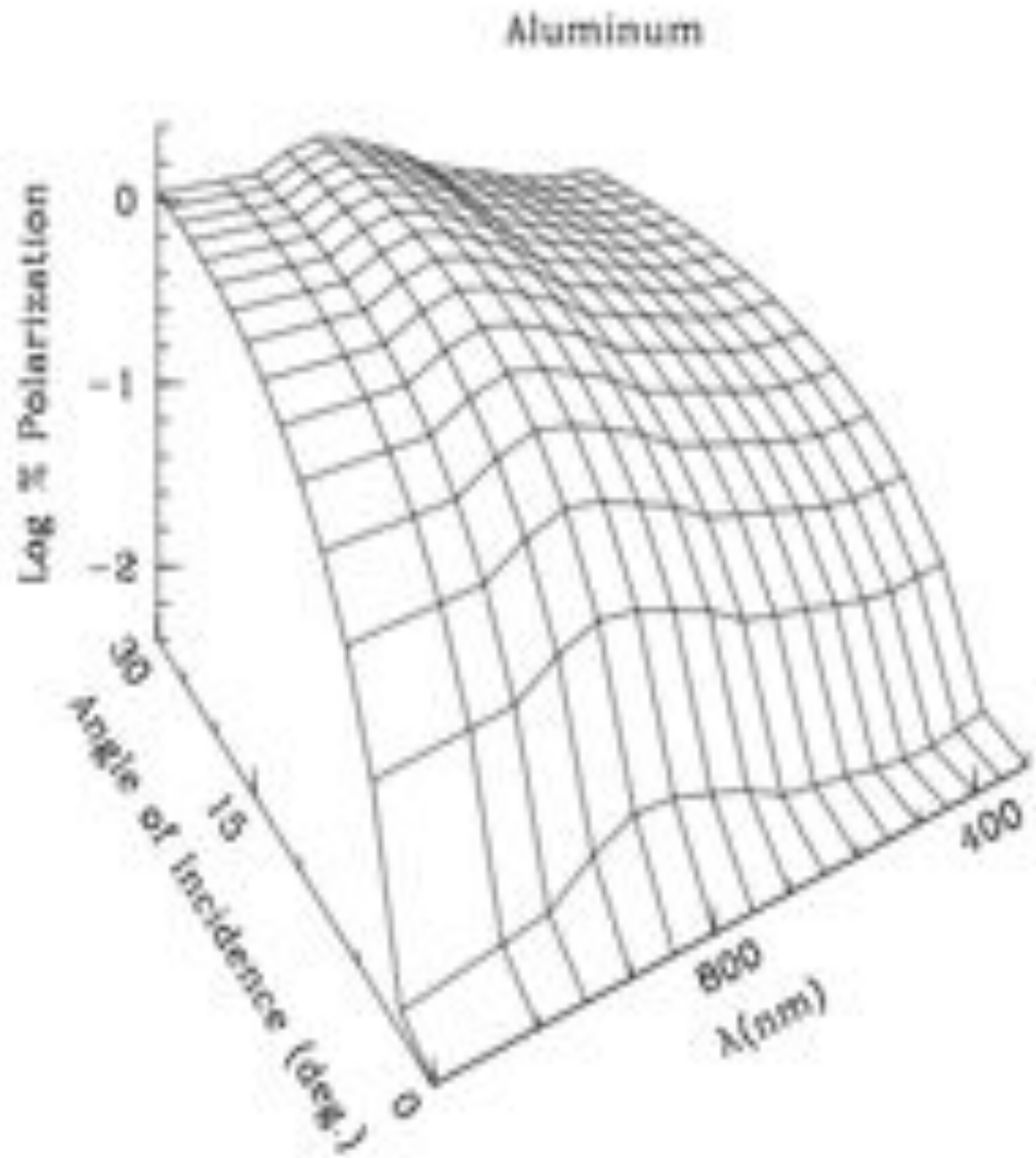
$$\theta_A > \theta_D$$

The angle of incidence for ray A at the flat mirror is about twice the angle of incidence for ray D at the flat mirror=> polarization content of the wavefront changes across the pupil

Aluminum polarization

% polarization

$$= \frac{R_{\parallel} - R_{\perp}}{R_{\parallel} + R_{\perp}}$$



Exit pupil transmittance $\left[c_2\left(\xi_2,\eta_2\right)+id_2\left(\xi_2,\eta_2;\lambda\right)\right]_T$

$c_2\left(\xi_2,\eta_2\right)$ Amplitude

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$d_2\left(\xi_2,\eta_2\right)$ Phase

- Mirror figure errors and core structure print through
- Polarization apodization
- **Optical thin film inhomogeneities**

Anisotropy polarization

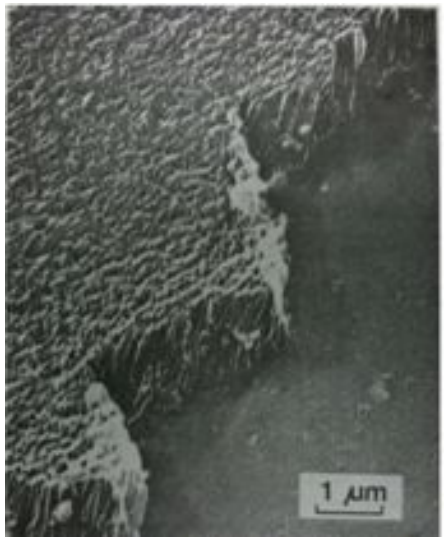
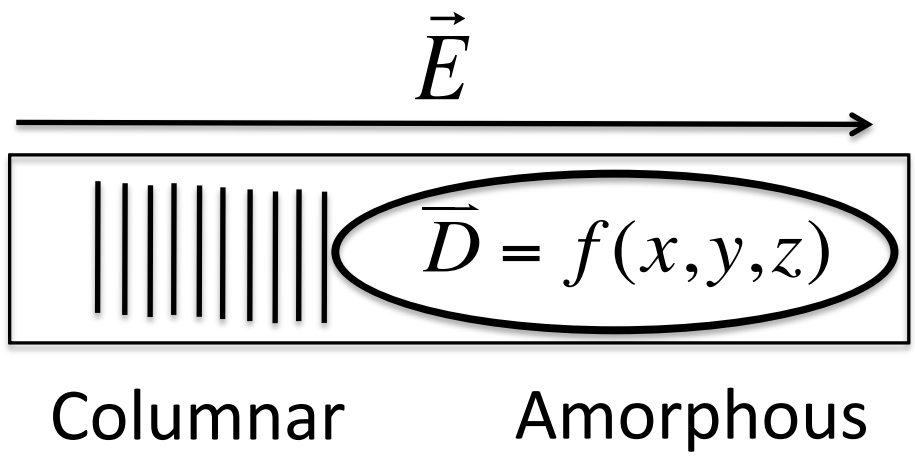


Image of the fractured edge of an Aluminum film showing inhomogeneities. Dirks & Leamy, Thin Solid Films **47**, 219-233 (1977)

Reflection results when *E* is incident and a virtual charge is conducted through the Al film causing the wavefront to be reflected.

Mirror coatings are inhomogeneous.

If the film is inhomogeneous then the film conductivity $\vec{D}$ inside the metal becomes a function of position (x,y). Wavefront errors occur.



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Thank you for your attention

Back up

Index of refraction on a highly reflecting thin, opaque aluminum film

Wavelength	n	k
nanometers		
918.41	1.860	8.440
953.73	1.470	8.950
991.88	1.370	9.490
1033.21	1.260	10.000
1078.13	1.210	10.600
1127.14	1.200	11.200
1180.81	1.210	11.800
1239.85	1.210	12.500
1305.11	1.230	13.200
1377.61	1.260	14.000
1458.65	1.330	14.900
1549.81	1.440	16.000
1653.14	1.590	17.100
1771.22	1.770	18.300
1907.46	1.990	19.800
2066.42	2.270	21.400
2254.28	2.620	23.300
2479.70	3.070	25.600

Compound reflections

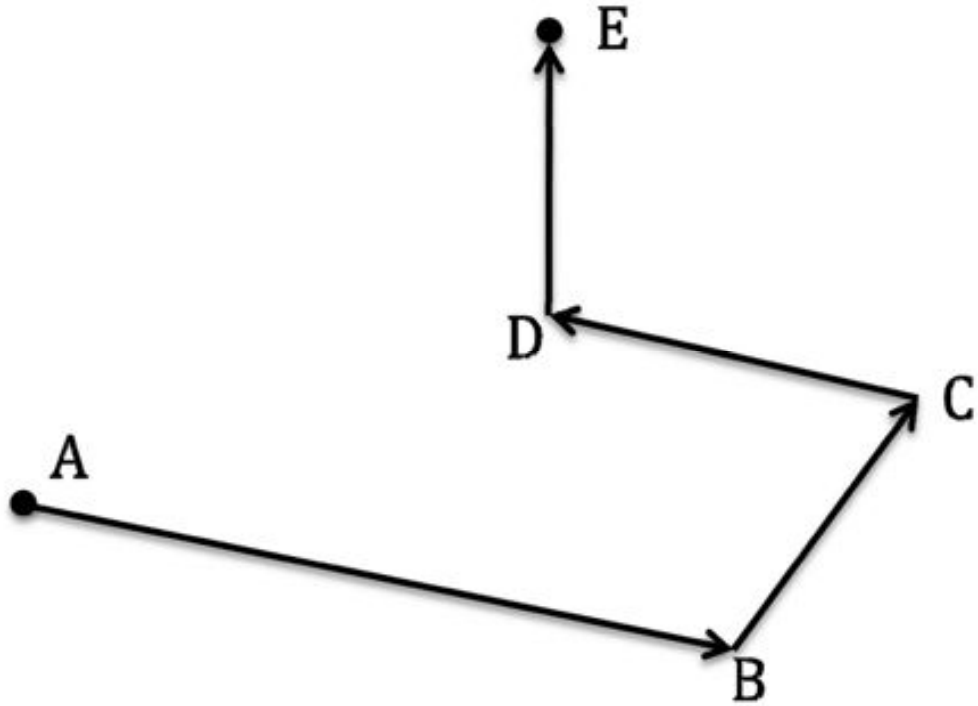

Figure 11. Diagram showing a beam of light starting at point A and traveling to point E after reflecting from three metal thin films on mirror substrates. The light makes two in-plane reflections at B and C and one out-of-plane reflection at D before traveling to point E. The reflections are all shown for 90-deg deviation. If the source A is white light and shows no preferential polarization, it will become partially linearly polarized upon reflection at B. The reflection at C will increase that partially linear polarization. The reflection at D will introduce a small amount of circular or elliptical polarization. The radiation that arrives at E is partially linearly polarized with some elliptical polarization component.

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If we use the parametric equations with u and v

- ....
- Hh
- Then .....

$$\hat{n}_2 \cos\theta_2 = u_2 + iv_2$$

$$\hat{n}_2 = n_2\left(1 + ik_2\right)$$

$$\left(u2 + jv2\right)^2 = \hat{n}_2^2 - n_1^2 \sin^2\theta_1$$

$$u_2^2 - v_2^2 = n_2^2\left(1 - k_2^2\right) - n_1^2 \sin^2\theta_1$$

$$and$$

$$u_2 v_2 = n_2^2 k_2$$

For light polarized parallel to the plane of incidence

$$\left[\rho_{1,2}\right]_{\parallel} =$$

$$\sqrt{\frac{\left[n_2^2\left(1-k_2^2\right)-n_1u_2\right]^2+\left[2n_2^2k_2\cos\theta_1-n_1v_2\right]^2}{\left[n_2^2\left(1-k_2^2\right)+n_1u_2\right]^2+\left[2n_2^2k_2\cos\theta_1+n_1v_2\right]^2}}$$

$$\tan\left[\phi_{1,2}\right]_{\parallel} =$$

$$2n_2^2k_2\cos\theta_1\left[\frac{2k_2u_2-\left(1-k_2^2\right)v_2}{n_2^4\left(1+k_2^2\right)\cos^2\theta_1-n_1^2\left(u_2^2+v_2^2\right)}\right]$$

For light polarized perpendicular to the plane of incidence

$$\tan\left[\phi_{1,2}\right]_{\perp} = \frac{2v_2 n_1 \cos\theta_1}{u_2^2 + v_2^2 - n_1^2 \cos^2\theta_1}$$

$$\left[\rho_{1,2}\right]_{\perp} = \sqrt{\frac{\left(n_1 \cos\theta_1 - u_2\right)^2 + v_2^2}{\left(n_1 \cos\theta_1 + u_2\right)^2 + v_2^2}}$$

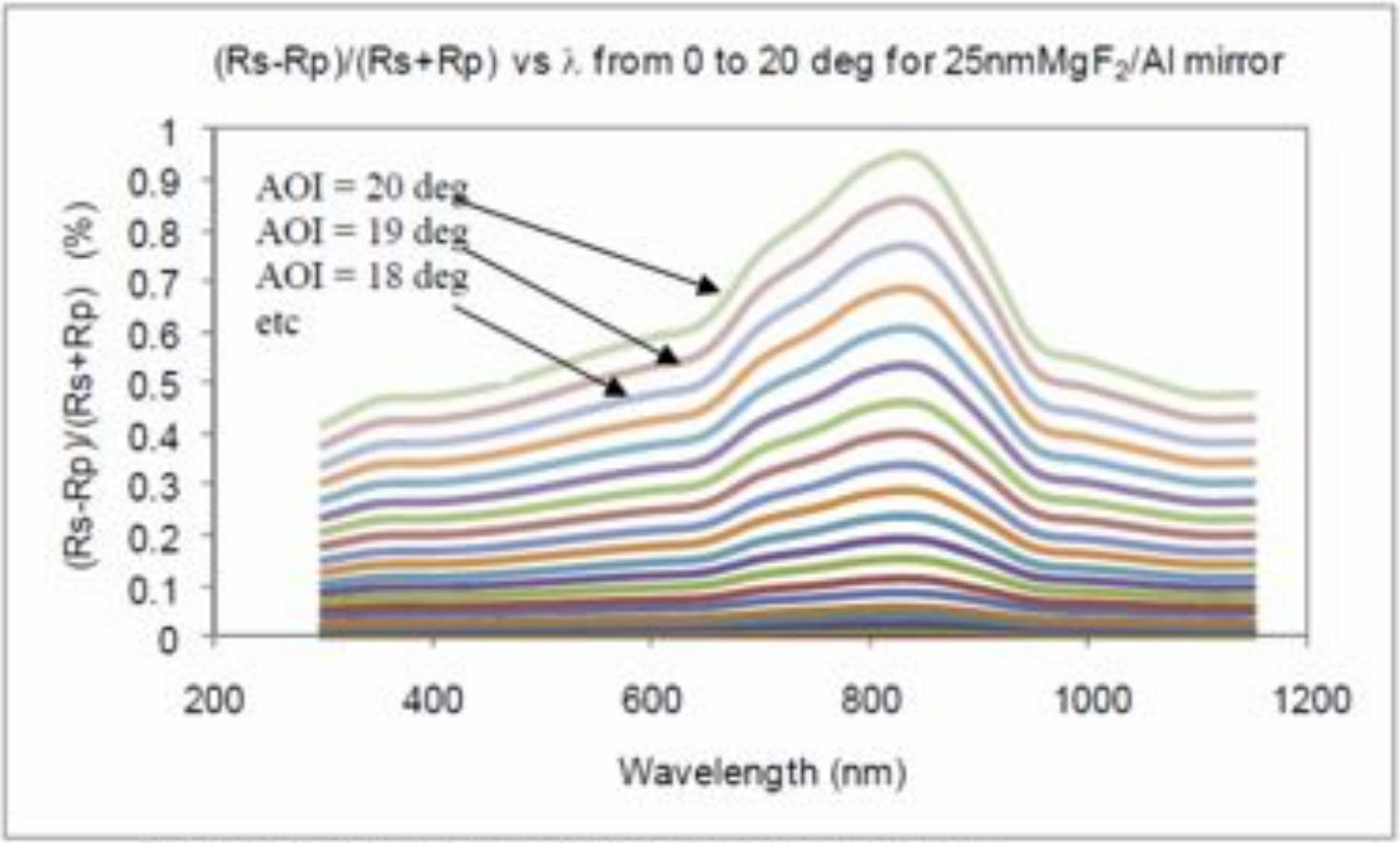

Figure 3. Polarization on reflection from an MgF_2/Al mirror

Kunjithapatham Balasubramanian et al.,
SPIE 8151paper # 81511G

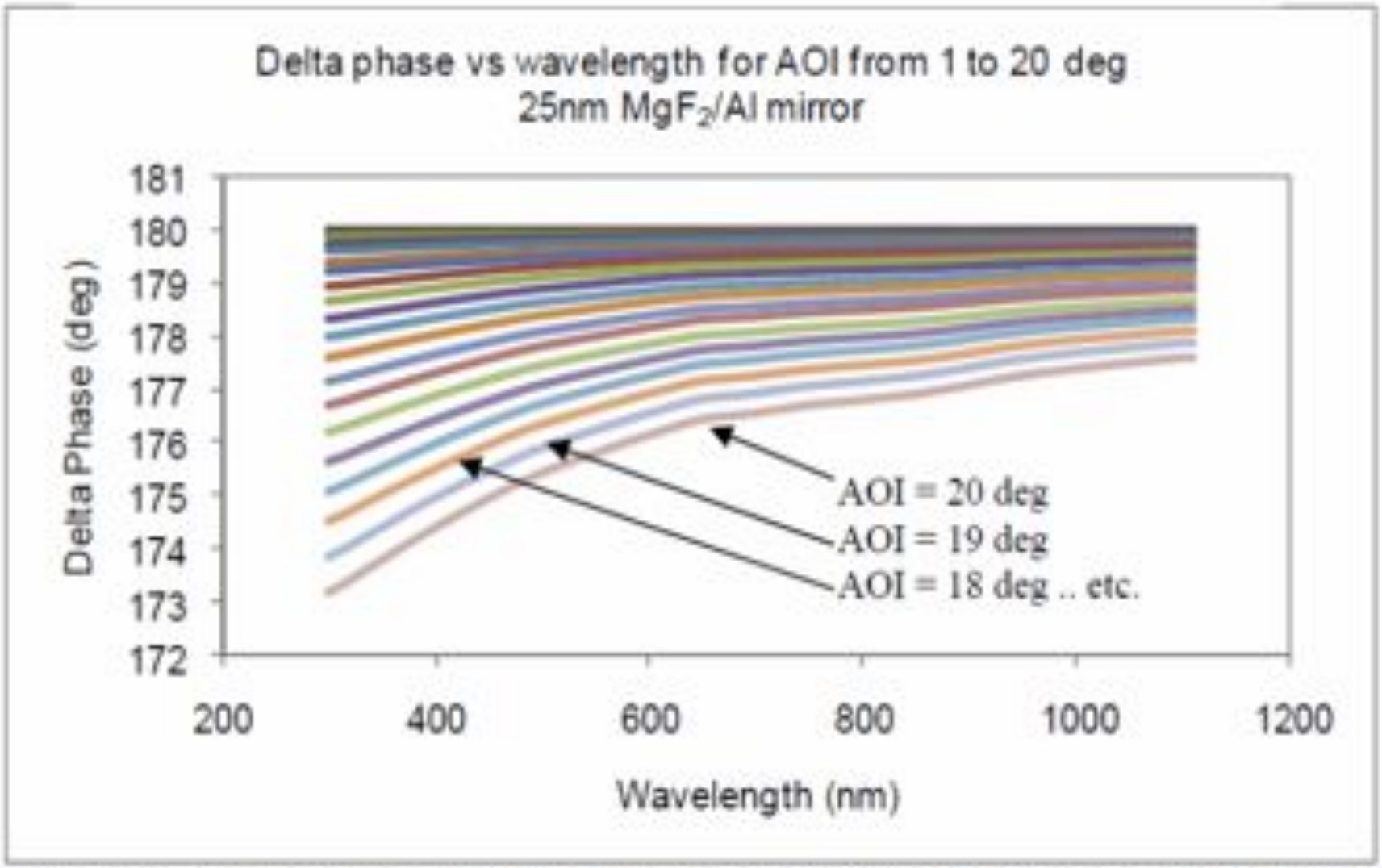

Figure 4. Delta phase between s and p polarization reflected off of 25nm MgF_2 protected aluminum mirror

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Phase

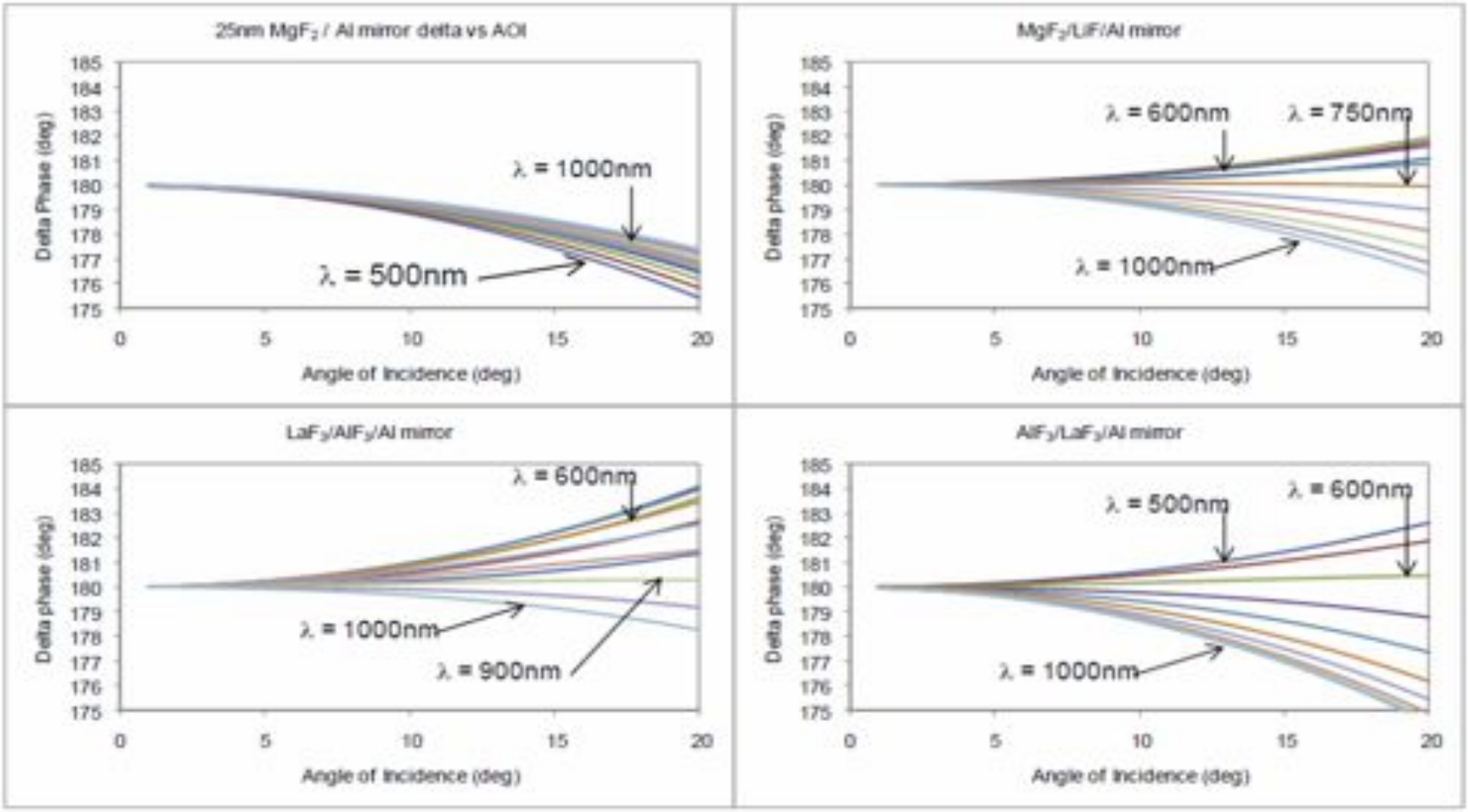

Figure 5. Phase difference between s and p polarizations as a function of AOI for various wavelengths from 500 to 1000 nm for different types of mirror coatings.

Kunjithapatham Balasubramanian et al.,
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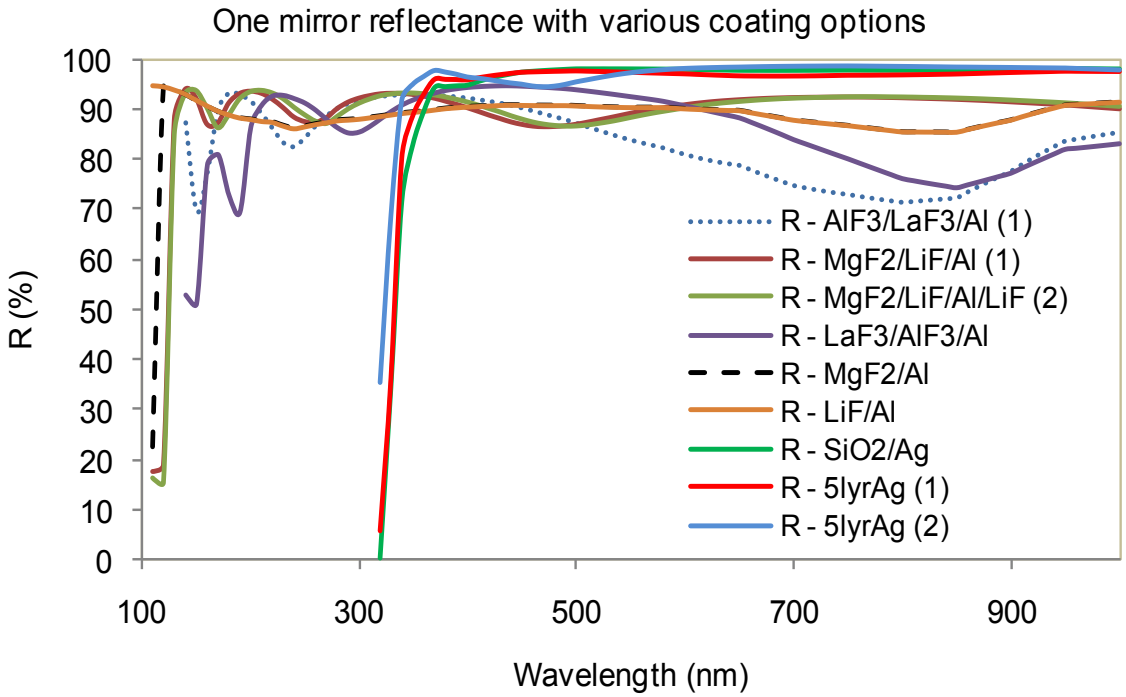

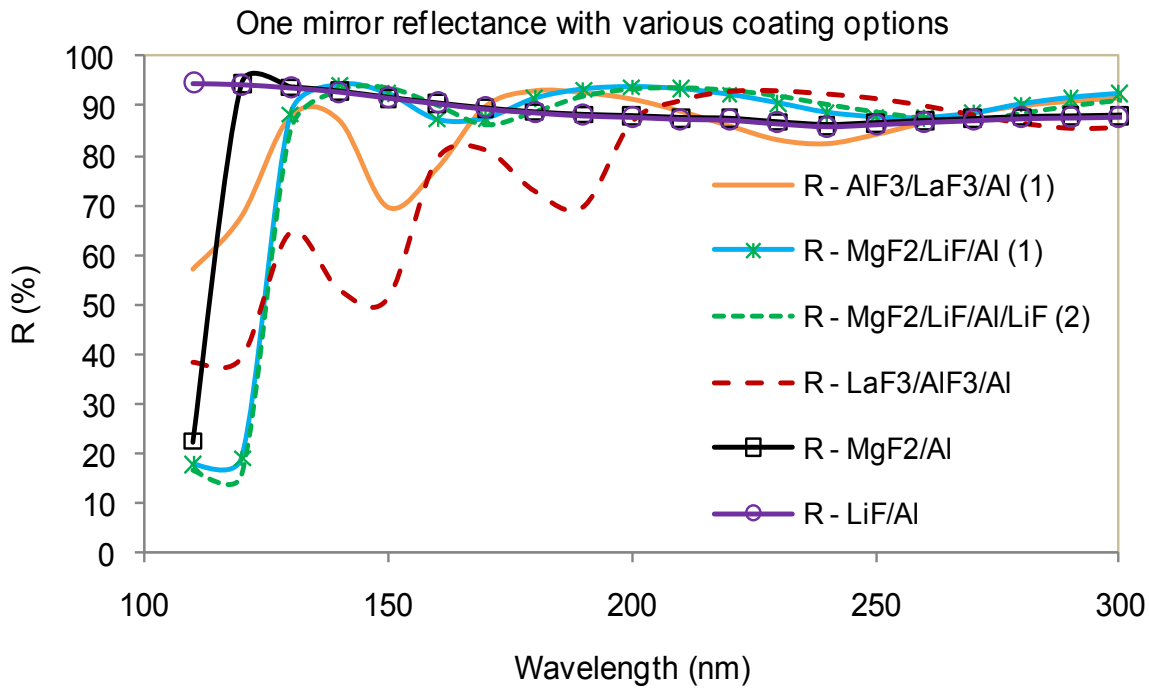

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